



RESEARCH MEMORANDUM

DYNAMIC INVESTIGATION OF TURBINE-PROPELLER

ENGINE UNDER ALTITUDE CONDITIONS

By Richard P. Krebs, Seymour C. Himmel Darnold Blivas, and Harold Shames

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SUMMARY

An investigation of the dynamics of a turbine-propeller engine was made in the NACA Lewis altitude wind tunnel employing the frequency-response technique for a range of pressure altitudes from 10,000 to 30,000 feet.

The investigation showed that the dynamic responses generalized for pressure altitude over the range of frequencies investigated. Further, the dynamic-response characteristics at any altitude could be predicted from steady-state-performance characteristics at one altitude.

A single run yielded the dynamic response of the enginepropeller combination and of the propeller alone. From these responses the dynamic response of the engine alone was synthesized.

The generalized time constants were found to be approximately 1.0 second for the engine-propeller combination, 0.36 second for the propeller alone, and 2.4 seconds for the engine alone.

INTRODUCTION

Accurate knowledge of the dynamic-response characteristics of gas-turbine-propeller engines and the factors that affect these characteristics are of great importance in the design of quick-acting, stable controls for this type engine. The term "dynamic response" refers to the manner in which the engine adjusts itself to changes in its independent variables and to the rapidity of these adjustments.

A theoretical analysis (reference 1) has related the dynamic response of the engine-propeller combination to the steady-state-performance characteristics and to the polar moment of inertia of



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the system. The analysis is extended herein to include a method of determining the dynamic response of the engine alone and of the propeller alone. Effects of altitude on the dynamic response are also indicated by theory (references 2 and 3), which affords a means of predicting the dynamic response at altitude from sea-level data.

In order to obtain experimental verification of these analyses, the steady-state-performance characteristics and the speed response of a turbine-propeller engine and its components to sinuscidally varying fuel flows over a range of pressure altitudes from 10,000 to 30,000 feet were investigated in the NACA Lewis altitude wind tunnel and are presented herein. A second purpose of the investigation is to demonstrate the process by which a third characteristic can be obtained by synthesis if any two of the dynamic characteristics (that of the engine alone, of the propeller alone, or of the engine-propeller combination) are known.

ANALYSIS

The behavior of the engine in progressing from one steady-state condition to another is described by the dynamic response of the engine. One of the most convenient ways of measuring dynamic response is by displacing the engine from its steady-state operation through the introduction of a specific disturbance into one of the independent variables. The applied disturbance instigates changes in the values of the engine dependent variables, and if certain of these variables are measured and analyzed the dynamic response of the engine can be determined. If the engine is a linear system, any type of applied disturbance should give the same dynamic response.

Forcing Function

Several techniques involving different disturbing functions and different analyses have been used in dynamic-response determinations. The step function has been found to be applicable in investigations involving radio, telephone and television amplifiers, and telephone transmission lines. This method has the advantages, especially in electrical work, of requiring very simple equipment to produce the forcing function and of requiring a minimum of time for the actual execution of the test. Qualitative results can be obtained without the application of extensive mathematics, but quantitative data, if the system has several energy storage units, require involved calculations or mechanical computers.

In mechanical or hydraulic systems, it is difficult to realize a true step input. In a previous unpublished investigation of a turbine-propeller engine, however, inputs in both blade angle and fuel flow, which approximated a step change, yielded satisfactory dynamic-characteristic data, with the help of a mechanical analyzer.

For the investigation described herein, the sinusoidal forcing function at several frequencies was used. The sinusoidal technique has the following advantages: First, because a sine wave passes through a linear system undistorted, the wave shape of all the engine variables is sinusoidal; accordingly, any component of the system can be analyzed by considering the variation of one variable as an input function and the variation of some other variable as an output. Second, the mathematics involved in determining the dynamics of a system from frequency-response data are very elementary.

The use of the sinusoidal technique to determine the dynamics of a system has the disadvantage that a large quantity of data must be taken. Obtaining the data and translating it into a usable and analyzable form is time-consuming and laborious. Furthermore, a complete description of the system dynamics requires a knowledge of both the amplitude response and the phase displacement as a function of the frequency of the applied sinusoidal forcing function.

Engine Equations and Time Constants

The analysis presented in this report is based on the following assumptions: At constant altitude and ram,

(1) Engine torque for small excursions from the steady-state point is a linear function of fuel flow and speed, or

$$Q_{\Theta} = Q_{\Theta} (W_{\widehat{\mathbf{1}}}, N)$$
 (B1)

(Definitions of the symbols used herein are given in appendix A.)

(2) Propeller torque for small excursions from the steady-state point is a linear function of blade angle and speed, or

$$Q_p = Q_p (\beta, N)$$
 (B2)

(3) The only energy-storage units are the engine and propeller inertias.

(4) The engine operates dynamically in a quasi-static state (reference 4).

From the foregoing assumptions and the mathematical development in appendix B, it follows that the response of engine speed to a change in either fuel flow or shaft torque is that of a firstorder lag system.

For constant engine fuel flow with a variation in blade angle producing the change in shaft torque, the response of engine speed to shaft torque is given by

$$\left(\frac{\Delta N}{\Delta Q_{\mathbf{B}}}\right)^{\mathbf{ML}} = \frac{\left(\frac{\partial N}{\partial Q_{\mathbf{B}}}\right)^{\mathbf{ML}} + \mathbf{I}_{\mathbf{B}}\mathbf{b}}{\left(\frac{\partial N}{\partial Q_{\mathbf{B}}}\right)^{\mathbf{ML}} + \mathbf{I}_{\mathbf{B}}\mathbf{b}}$$
(B8)

Regardless of the type of experimental technique used, dynamic response for a system defined by an equation similar to equation (B8) can be described by a characteristic parameter, the time constant of the system. The time constant may be variously described as the time required for the response to reach 0.63 of its final value with a step function imposed on the system, the reciprocal of the frequency at which the response falls to 0.707 of its zero-frequency value with a sinusoidal variation applied to the system, or the reciprocal of the frequency at which the phase angle between the sinusoidal forcing function and the response is 45° (corner or break frequency). The synonymity of these definitions for the time constant has been frequently indicated in the literature (references 5 and 6).

The time constant in the response given by equation (B8) is

The quantities on the right-hand side of equations (B8) and (B9) are concerned only with the engine. The dynamic response and the time constant for the engine alone are similar to those for a turbojet engine (reference 3).

When the blade angle is held constant and a varying fuel flow is applied to the engine, the engine produces a varying shaft torque. The dynamic response of propeller speed to shaft torque is then a function of the propeller alone. The response is given by

$$\left(\frac{\Delta N}{\Delta Q_B}\right)_{\beta} = \frac{1}{\left(\frac{\partial Q_p}{\partial N}\right)_{\beta} + I_p p}$$
(B12)

The corresponding time constant is

$$\tau_{\mathbf{p}} = \frac{1_{\mathbf{p}}}{\left(\frac{\partial Q_{\mathbf{p}}}{\partial N}\right)_{\beta}} \tag{B13}$$

The response of engine speed to fuel flow at constant blade angle is given by

$$\left(\frac{\nabla M^{L}}{\nabla M}\right)^{\beta} = \frac{\left(\frac{\partial M}{\partial G^{\Theta}}\right)^{M-L} + \left(\frac{\partial M}{\partial G^{\Theta}}\right)^{\beta} + (I^{\Theta} + I^{D})^{D}}{\left(\frac{\partial M^{L}}{\partial G^{\Theta}}\right)^{M}}$$
(B12)

The associated time constant is

$$\tau_{c} = \frac{\left(\frac{\partial Q_{e}}{\partial N}\right)_{W_{e}} + \left(\frac{\partial Q_{p}}{\partial N}\right)_{\beta}}{\left(\frac{\partial Q_{p}}{\partial N}\right)_{\beta}}$$
(B16)

Further, equation (B15) shows how the engine and propeller characteristics combine to give the engine-propeller characteristic (speed - fuel-flow response). If either the engine or the propeller is used in a different combination, that part of the dynamic response of the new combination contributed by the original engine or propeller will be unchanged.

The response of the engine, the propeller, or the enginepropeller combination may be determined if any two of the dynamic characteristics are known (appendix B). A synthesized dynamic response for the engine may be computed from the equation

$$\left(\frac{\Delta N}{\Delta Q_{\rm g}}\right)_{\rm W_{\rm f}} = \left(\frac{\partial Q_{\rm e}}{\partial N}\right)_{\rm W_{\rm f}} + \left\{\left[\left(\frac{\partial Q_{\rm p}}{\partial N}\right)_{\beta} + \left(\frac{\partial Q_{\rm e}}{\partial N}\right)_{\rm W_{\rm f}}\right]^{\rm T}_{\rm c} - \left(\frac{\partial Q_{\rm p}}{\partial N}\right)_{\beta}^{\rm T}_{\rm p}\right\}_{\rm p} \tag{1}$$

and the synthesized time constant is

$$\tau_{e} = \frac{\left[\left(\frac{\partial Q_{p}}{\partial N}\right)_{\beta} + \left(\frac{\partial Q_{e}}{\partial N}\right)_{W_{f}}\right] \tau_{c} - \left(\frac{\partial Q_{p}}{\partial N}\right)_{\beta} \tau_{p}}{\left(\frac{\partial Q_{e}}{\partial N}\right)_{W_{f}}}$$
(B28)

Equations (B8), (B9), (B12), (B13), (B15), (B16), (1), and (B28) may be presented in generalized form by multiplying each variable by the appropriate correction factor as shown in appendix B.

For example, the time constant for the engine as defined by equation (B9) becomes

$$\tau_{e} \frac{\delta}{\sqrt{\theta}} = \frac{I_{e}}{\left(\frac{\partial Q_{e}/\delta}{\partial N/\sqrt{\theta}}\right)_{W_{f,corr}}}$$
(B18)

in which the quantity on the left is referred to as the "corrected time constant" and $W_{f,corr}$ is the corrected fuel flow.

This correction, although applied to a specific time, namely the time constant, can be applied to time in general. Because frequency is a function of time it is similarly generalized to a corrected frequency $\omega\sqrt{\theta}/\delta$.

It should be noted that the corrected time and corrected frequency just described are useful in describing the responses of systems involving only aerodynamic and inertia forces.

APPARATUS AND INSTRUMENTATION

Wind-Tunnel Installation

The turbine-propeller engine investigated was installed in a wing supported in the 20-foot-diameter test section of the NACA Lewis altitude wind tunnel (fig. 1) where it was possible to subject the engine to simulated conditions of altitude pressure, temperature, and flight speed.

Engine and Propeller

The engine has a nominal rating of 3670 shaft-horsepower and 1150 pounds of jet thrust under static sea-level conditions at an engine speed of 8000 rpm and a tail-pipe gas temperature of 1094° F.

This engine, which is schematically shown in figure 2, has a 14-stage axial-flow compressor directly coupled to a two-stage turbine. An eight-blade, counter-rotating, hydraulically operated propeller is driven by a gear train containing a hydraulic torquemeter.

Air entering an annular inlet, located well back of the propeller, is turned through 180° and flows forward through the compressor. Upon leaving the compressor, the air is again turned 180° and flows through 11 can-type combustion chambers radially located around the compressor casing. After burning, the hot gases pass through the two-stage turbine and out the fixed-area tail pipe.

Fuel System

For the dynamic investigation with variable fuel flow, fuel was supplied to the engine by an auxiliary fuel pump. A schematic diagram of the fuel system used is shown in figure 3. The fuel flowed from the pump through a throttling valve to the fuel distributor where the main stream was divided into 11 parts and distributed to each of the combustion chambers. Each combustion chamber contained a fuel vaporizer, which operated on a comparatively low fuel pressure. The throttling valve was used to decrease the fuel-distributor pressure at high altitudes and yet maintain a high pump operating pressure.

The fuel-pump-outlet pressure was determined by an external, variable control-oil pressure and is linearly related to this pressure (reference 7). Thus, sinusoidal fuel pressures were achieved

by impressing a sinusoidal variable control-oil pressure on the pump. An accumulator introduced the predetermined base control oil pressure, and an hydraulic sine-wave generator consisting of a circular cam driving a piston in an hydraulic cylinder imposed a sinusoidal pressure on the base pressure (fig. 3). Variable frequency was obtained by using a continuously-variable-speed transmission. Variable amplitudes were obtained by using a phase-changing mechanism and a flow-control valve. A detailed description of the pump and sine-wave generator is presented in reference 7. The time constant of the pump (at the speed at which it was operated) is 0.03 second.

Instrumentation

Measurement of engine variables for dynamic testing of the engine requires fast-responding instrumentation in order to obviate the necessity of correcting the data for dynamic errors introduced by the instruments. Further, instruments of high sensitivity are required to measure the small changes in the engine variables during dynamic testing. The nature of these highly sensitive, fast-responding instruments is such that absolute values of the variables measured must be obtained by calibrating the dynamic instruments against accurate but slower-responding instruments at steady-state points for each given run. For these reasons two sets of instruments were used, one set for steady-state operation and calibration purposes and the other set for dynamic measurements, with response times that are fast in comparison to that anticipated for the engine.

Steady-state instrumentation. - For steady-state operation, pressures of the working fluid were measured by water and mercury manometers and were photographically recorded. Temperatures were obtained by iron-constantan and chromel-alumel thermocouples and were recorded by self-balancing potentiometers. The location of the various instrumentation stations throughout the engine are shown in figure 2. The following other engine variables were measured: Engine speed was measured by a stroboscopic tachometer, torque by a piston-type torquemeter located in the gear train, torquemeter pressure, fuel-distributor pressure, and fuel-pump control-oil pressure by Bourdon gages, and fuel flow by rotameters.

Propeller pitch was determined by a pitch indicator attached to one of the blades of the rear propeller. The pitch indicator consisted of an accurately wound slide wire, rigidly attached to the shank of the blade, the wire being an element of a potentiometer circuit. A spring-loaded brush attached to a flange on the propeller assembly rode on the slide wire as the wire moved relative to the

brush. The signals thus obtained and the voltage across the slide wire were carried to a nonrotating point through three slip rings located on the rear diaphragm of the propeller assembly. This signal was then sent to a microammeter, which served as the indicating device.

Dynamic instrumentation. - For dynamic operation of the engine the following instrumentation was used:

Torquemeter pressure and fuel-pump control-oil pressure were measured by strip-chart pressure recorders whose sensing elements were Bourdon tubes. These Bourdon tubes in the recorders have a flat frequency response well past 6 radians per second. All the tubing that led to the sensing points and the Bourdon tubes were filled with oil.

In order to maintain accuracy and sensitivity over the range of altitudes encountered, two sets of instruments were used for sensing these pressures; one set was used for altitudes below 20,000 feet and the other set for 20,000 feet and above. Records of typical torque and variable control-oil pressure traces are shown in figures 4(a) and 4(b).

Engine speed was sensed by a direct-current tachometer driven off one of the accessory pads of the engine. The tachometer output was sent through a low pass filter to attenuate the extraneous signals and thence to the oscillograph. The time constant of the complete circuit was less than 0.05 second.

Though not used in this report, compressor-outlet temperature (station 2) and tail-pipe temperature (station 5) were sensed by special chromel-constantan thermocouples installed at these points. These thermocouple signals were fed directly into the oscillograph for recording. Certain pressures within the engine and the tunnel-air-stream dynamic pressure were measured with strain-gage pressure transducers whose outputs were recorded by the oscillograph.

All measurements in the form of electrical signals were recorded on a multichannel galvanometric oscillograph. The elements of the oscillograph had a natural frequency of 40 cycles per second and the input circuits were designed to critically damp these elements. A typical record from this instrument is shown in figure 4(c).

PROCEDURE

In order to determine the dynamic characteristics of the propeller and the engine-propeller combination, sinusoidal variations at various frequencies were applied to the engine fuel flow, with the engine operating at pressure altitudes of 10,000, 20,000, and 30,000 feet, with an engine-cowl-inlet temperature of approximately 10° F and a tunnel Mach number of 0.24.

The mean condition for the sinusoidal investigation was set at 7600 rpm (the maximum continuous-cruising speed of the engine) and a tail-pipe temperature of approximately 875° F (estimated to be 20° F below the compressor stall limit at that speed). These settings were made by independently adjusting both fuel flow and blade angle.

At the mean condition for the dynamic investigation, the bladeangle controller was locked and the sine-wave generator was brought
into operation. The amplitude of the input fuel-flow sine wave was
adjusted to approximately the same increment as corresponded to a
100 rpm change during steady-state operation. Before the dynamic
records of several cycles were taken, sufficient time was allowed
to elapse, with the engine running sinusoidally, to permit any
transients in the sine wave to die out. The procedure was repeated
for a number of frequencies between 0.2 and 6.0 radians per second.
A series of frequency-response runs at several input amplitudes
was also made at a pressure altitude of 20,000 feet. The dynamic
instruments were calibrated at the beginning and the end of each
series of frequency runs by recording steady-state data at several
operating points about the mean operating condition for the sinusoidal runs on both dynamic and steady-state instruments.

Steady-state performance maps were determined from constant engine-speed and constant blade-angle runs at the three altitudes.

PRESENTATION OF DATA

In presenting the results of the application of the sinuscidal technique for the determination of dynamic responses, the data are shown in plots of amplitude response and phase angle as a function of the frequency and corrected frequency of the applied inputs. For this particular investigation, the dynamic responses of engine speed to fuel flow and engine speed to shaft torque, measured in the gear train between engine and propeller, at constant blade angle are considered. It has been shown that these two responses give the dynamic characteristics of the engine-propeller combination and of the propeller alone.

The amplitude response to a sinusoidal forcing function is given as the magnitude of the change in a dependent quantity, designated the output, divided by the corresponding change in the forcing function, designated the input. Because the dynamic characteristic depends on the relative change of the amplitude response with frequency, the data in this report have been normalized by dividing the amplitude response by its value at zero frequency, which brings the response to unity at low frequencies.

In presenting dynamic-response data in this report, the engine fuel flow has been assumed to be linearly related to the variable control-oil pressure of the fuel pump for small amplitude fluctuations of the variable control-oil pressure. A typical relation between changes in variable control-oil pressure and the associated changes in engine fuel flow is shown in figure 5. The range of variable control-oil pressure shown is greater than those changes encountered during the sinusoidal investigation. The linear relation in the magnitudes is apparent. The corner frequency of the pump was more than five times the highest frequency impressed on the engine (reference 7) and therefore the dynamics introduced by the pump have negligible effect on the amplitude response of the system but slightly affect the phase response.

RESULTS

In accordance with the ANALYSIS, the dynamic responses of the engine, the propeller, and the engine-propeller combination are predicted from the slopes of the steady-state speed-torque curves. Experimentally, the dynamic responses of the propeller and the engine-propeller combination are obtained from the constant-blade-angle, sinusoidal fuel-flow investigation. By a process of synthesis, the dynamic response of the engine alone is obtained from the results of the constant-blade-angle dynamic investigation. The dynamic responses obtained from the sinusoidal fuel-flow investigation are compared with those predicted from the slopes of the steady-state performance curves.

Steady-State Characteristics

Engine. - The engine steady-state characteristics are shown in figures 6 and 7 wherein curves of corrected torque and corrected tail-pipe temperature, respectively, are plotted as functions of corrected engine fuel flow for constant corrected engine speeds. Data were taken at simulated altitudes of 10,000, 20,000, and 30,000 feet and a free-stream Mach number of 0.24.

With reference to the ANALYSIS it can be seen that to determine the dynamic response of the engine at a particular operating condition it is necessary to know the slope of the torque-speed constant-fuel-flow curve at this operating point. For the dynamic runs, the basic operating point was determined by a choice of engine speed and tail-pipe temperature; the values selected were 7600 rpm and approximately 875° F, respectively.

In order to obtain the torque-speed curves at constant fuel flow at the mean condition for the dynamic runs, the following procedure was used: Cross plots from the corrected tail-pipe temperature - corrected fuel-flow curves (fig. 7) were made for the specific values of corrected tail-pipe temperature that prevailed at the mean condition (shown in the subsequent table) for the sinusoidal runs at the three pressure altitudes investigated (fig. 8). The corresponding mean corrected fuel flows were determined by entering this plot (fig. 8) at the appropriate corrected mean engine speed for the three altitudes. Cross plots of the curves of figure 6 were then made for the values of fuel flows thus determined to give the torque-speed curves of figure 9.

The corrected torque and the corrected tail-pipe temperature do not have the same value for a given corrected engine speed and fuel flow at all altitudes (figs. 6 and 7). For a limited range of engine speed, however, the curves of figure 9 indicate that the slopes of the torque-speed curves at a given corrected speed and nearly identical corrected fuel flows have approximately the same value for pressure altitudes between 10,000 and 30,000 feet. The generalized dynamic response predicted from the slopes of the steady-state curves will therefore be nearly the same for all altitudes investigated. Further, this similarity of dynamic responses has resulted from an engine the basic parameters of which fail to generalize for the range of altitudes investigated.

The slopes of the constant fuel-flow curves (fig. 9) and the resultant characteristic of the engine dynamic response, the corrected time constants as determined by equation (B18) of appendix B, are given in the following table:

Pressure altitude (ft)	Mean corrected tail-pipe temperature (^C R)	Mean corrected engine speed (rpm)	Mean corrected fuel flow (lb/kr)	Slope of steedy-state curve (20/6) Wf,corr (1b-ft-sec)	Corrected time constant $\tau_{\theta} \frac{\delta}{\sqrt{\theta}}$ (sec)
10,000	1504	7960	3020	3.3	2.4
20,000	1487	8000	2925	3.2	2.5
30,000	1456	8020	2880	3,3	2.4

These corrected time constants for the engine alone generalize within the accuracy attributed to the data. This correlation indicates that the corrected engine dynamic responses as predicted from the slopes of the steady-state curves are nearly constant for the altitudes considered.

<u>Propeller.</u> - The propeller steady-state characteristics are shown in figure 10 wherein corrected torque is plotted as a function of corrected engine speed at constant blade angles. Data were taken at pressure altitudes of 10,000, 20,000, and 30,000 feet at an engine inlet-air temperature of approximately 10° F and a free-stream Mach number of 0.24.

The blade angles for the curves of figure 10 were chosen so that at an engine speed of 7600 rpm the tail-pipe temperature would be approximately 875° F. It should be noted that under these conditions neither the propeller-blade angle nor the corrected torque has the same value at all altitudes for a given value of corrected engine speed. For a considerable range of corrected engine speed, however, the curves of figure 10 indicate that the slopes at a given engine speed are nearly the same for altitudes between 10,000 and 30,000 feet. Therefore, as for the engine alone, generalization of the dynamic response of the propeller alone is predicted.

The slopes of the constant-blade-angle curves (fig. 10) and the resultant corrected propeller time constants as determined by equation (B20) of appendix B at the three pressure altitudes are given in the following table:

Pressure altitude (ft)		Mean corrected tail-pipe temperature (°R)	Mean corrected engine speed (rpm)	Slope of steady-state curve $\left(\frac{\partial Q/\delta}{\partial N/\sqrt{\theta}}\right)_{\beta}$ (lb-ft-sec)	Corrected time constant $\tau_p \frac{\delta}{\sqrt{\theta}}$ (sec)
10,000	36.5	1504	7960	7.4	0.36
20,000	35.4	1487	8000	7.5	•35
30,000	34.3	·1456	8020	7.4	.36

These corrected time constants for the propeller alone generalize within the accuracy of the data, implying good correlation of the corrected dynamic responses of the propeller for the range of altitudes investigated.

Engine-propeller combination. - The dynamic response of the engine-propeller combination can be computed from the combined slopes of the torque-speed curves at constant blade angle and constant fuel flow. The sum of the two slopes and the resultant corrected time constants, characteristic of the dynamic responses, at the three pressure altitudes are given in the following table:

Pressure altitude (ft)	Summation of slopes of steady-state curves $\left(\frac{\partial Q/\delta}{\partial N/V\theta}\right)_{W_{f,corr}} + \left(\frac{\partial Q/\delta}{\partial N/V\theta}\right)_{\beta}$	Corrected time constant $\tau_{c} \frac{\delta}{\sqrt{\theta}}$
	(lb-ft-sec)	(sec)
10,000	10.7	0.99
20,000	10.7	.99
30,000	10.7	•99

As would be expected, because the dynamic responses of both the engine and propeller very nearly generalized for the pressure altitudes investigated, those responses of the combination likewise generalized.

Linearity Investigation

One of the assumptions made in the ANALYSIS for the dynamic response was that the engine, the propeller, and the engine-propeller combination could be treated as linear systems over a limited range of the variables. Two of the functions that determine the dynamic response of the engine, the propeller, or the combination are the slopes of the torque-speed curves (figs. 9 and 10). These slopes do not change appreciably within a limited range of operating conditions. The assumption of linearity is therefore validated in the steady state and the possibility of dynamic response linearity is predicted.

When the engine is passing from one steady-state condition to another, however, or when it is operating in some condition other than equilibrium, it is possible that certain dynamic phenomena may be present that would introduce nonlinearities into the dynamic response. In order to investigate the possibility of dynamic response nonlinearity, frequency-response runs at three different fuel-flow amplitudes at constant blade angle were made at a pressure altitude of 20,000 feet. The amplitude response of the engine-propeller combination for these runs is shown in figure 11. The curve represents the average of the data points. The data points shown are for constant-amplitude sinusoidal variations in fuel flow of 100, 180, and 300 pounds per hour peak to peak. The distribution of the data points about this mean curve shows no consistent trend with increasing input amplitude up to 300 pounds per hour, which corresponds to a speed change at zero frequency of 500 rpm peak to peak and is approximately 20 percent of the engine operating speed range. Therefore, within this range the enginepropeller combination may be considered free from any gross dynamic nonlinearity.

Dynamic Characteristics

Engine-propeller combination. - The ANALYSIS shows that the dynamic response of the engine-propeller combination can be obtained from the constant-blade-angle, variable-fuel-flow dynamic investigation. The results pertinent to the determination of engine-propeller-combination dynamics are shown in figure 12. The amplitude and the phase response at pressure altitudes of 10,000, 20,000, and 30,000 feet at a base engine speed of 7600 rpm and a tail-pipe temperature of approximately 875° F are shown in figure 12. The amplitude data shown for a pressure altitude of 20,000 feet are the averages of the data from the linearity investigation; phase-angle

data were not obtained for these runs. The first-order lag system predicted by the ANALYSIS is borne out by the amplitude-response data of figure 12(a), which is primarily the response of such a system. The phase responses of figure 12(b) show a phase lag in excess of 90°. These phase angles greater than those of a first-order-lag system are considered to arise from lags in the engine fuel system, possible dead time in instrumentation lines, or both. The data of figure 12(b) show the phase angles that would have to be considered in the design of a control in which the variable-control oil pressure would be regulated by the output of the engine tachometer. The locus diagram for the engine-propeller combination is shown in figure 12(c).

The corner frequencies of the primary first-order responses, as determined from the amplitude response curves and the corresponding time constants for the engine-propeller combination are given in the following table:

Pressure altitude (ft)	Corner frequency (rad/sec)	Time constant ^T c (sec)	Corrected time constant $\frac{\delta}{\sqrt{\theta}}$ (sec)
10,000	0.74	1.4	1.00
20,000	.47	2.1	1.00
30,000	.32	3.1	.97

The data of figure 12 are replotted in figure 13 using the corrected frequency developed in the ANALYSIS. The dynamic responses generalize for altitude throughout the corrected frequency range investigated within the accuracy of the data. The corrected corner frequency and the corresponding corrected time constant of the primary first-order lag for the engine-propeller combination, as determined from the amplitude response curve of figure 13, are 1.01 radians per second and 0.99-second, respectively.

Propeller. - The ANALYSIS section shows that the dynamic response of the propeller can be obtained from the constant blade-angle, variable fuel-flow dynamic investigation by considering the torque as the input and the speed as the output of the system. The propeller is considered to be a system having only acrodynamic and inertia forces acting on it and the propeller is shown to be a first-order lag system.

The experimentally determined dynamic response of the propeller is shown in figures 14 and 15 as the amplitude and phase response of engine speed to shaft torque. Shown in figure 14(a) are solid curves through the data points and, in view of the ANALYSIS, dashed firstorder curves that best fit the data points. The solid curves depart from the dashed curves as though there were an additional lead in the The phase response is in agreement with the amplitude data. The dynamics of the propeller are involved in the response of the engine-propeller combination; because a gas-turbine engine alone has been shown to be primarily a first-order lag system (reference 3), it can be analytically shown that the propeller must be a first-order lag to obtain a first-order lag response for the combination. In the speed-torque relation for the propeller response (fig. 14(a)), the speed data used is the same as that used to determine the response of the combination; therefore, the most plausible explanation for the divergence of the response data from the first-order system was a lag in the measured torque that would manifest itself as a lead in the propeller dynamic response.

From the foregoing considerations, the primary first-order lag is considered to be due to the propeller; the corner frequencies and corresponding time constants for the three altitudes are given in the following table:

Pressure altitude (ft)	Corner frequency (rad/sec)	Time constant ^T p (sec)	Corrected time constant $\tau_{p} \frac{\delta}{\sqrt{\theta}}$ (sec)
10,000	1.75	0.57	0.41
20,000	1.40	.71	•35
30,000	.90	1.11	.35

The data of figure 14 are replotted against corrected frequency in figure 15. These data have been treated in a manner similar to that used for the data of figure 14. The lag of the propeller dynamics for the three altitudes merge on the amplitude plot (fig. 15(a)) to form a single corrected lag with a corrected corner frequency of 2.73 radians per second corresponding to a corrected time constant of 0.37 second.

DISCUSSION OF RESULTS

The dynamic responses obtained from the steady-state slopes and those given by the dynamic runs are compared in the following table, wherein the comparison is placed on a quantitative basis by consideration of the time constants obtained from the steady-state curves and time constants of the primary first-order lags found from the dynamic runs:

			D,	ynamio ru	ns	Steady-state runs	3
	Pressure altitude (ft)	Correction factor	Corner frequency (rad/sec)		Corrected time constant T $\frac{\delta}{\sqrt{6}}$ (sec)	(8/8) (8/8)	Corrected time constant $\tau \frac{\delta}{\sqrt{\theta}}$ (sec)
Engine- propeller combination	10,000 20,000	0.72	0.74	2.1	1.00	10.7 10.7	0.99
	30,000	.31	.32	5.1	.97	10.7	.99
				-		(30/8) ∂ a /√€) _β	
Propeller	10,000	0.72	1.75	0.57	0.41	7.4	0.36
	20,000	.48	1.40	.71	.35	7.5	.35
_	30,000	.31	.90	1.11	.35	7.4	.36

This table shows the following results: increase in time constant with pressure altitude, reduction of the time constants to a single value for a range of altitudes by means of the correction factor $\delta/\sqrt{6}$, and good agreement between the time constants computed from steady-state slopes and those measured by the dynamic investigation.

Analytically, the change in the dynamic response with altitude is explained by equations (B8), (B12), and (B15) by a reduction of the slopes of the torque-speed curves. Physically, the slower-acting system can be explained by the decrease in available torque at

altitude due to the reduction of air density and a consequent reduction in the change of torque for a given change of engine speed. The polar moment of inertia, being a function of the dimensions and materials of the rotating parts, remains constant. Consequently, with a constant moment of inertia and a reduced increment in the aerodynamic forces as altitude increases, a slower-responding system results. Quantitatively, the time constants have a range of 1:2 over the range of pressure altitudes from 10,000 to 30,000 feet.

The effect of air pressure and temperature is shown by modifying the time constant by the factor $\delta/\sqrt{\theta}$ to give a corrected time constant $^{T}\frac{\delta}{\sqrt{\theta}}$, which the theory shows to be invariant with altitude. The application of this factor to the measured time constants brings them into close agreement for the altitudes investigated. This agreement of the corrected time constants is considered to be within the accuracy of the data.

The values of corrected time constants determined from the dynamic investigation and those calculated from the steady-state data agree, as can be observed from the preceding table. The average value of the corrected time constant determined from the dynamic investigation for the engine-propeller combination is 0.99 second, which is the same as that found by the steady-state-slope method, and the corrected time constants attributed to the propeller are 0.37 and 0.36 second, respectively. The data of this report bear out the theory that the dynamic responses at altitude can be predicted from sea-level-performance tests.

For a turbine-propeller engine, the following dynamic responses are of major interest for control purposes: the engine alone, the propeller alone, and the engine-propeller combination.

If the moments of inertia of the components are known, the dynamic responses may be determined from the steady-state characteristics. If the moments of inertia are not known, the response of the components and the combination may be determined by synthesis from the steady-state performance characteristics and a single dynamic run employing the technique of frequency-response testing.

The corrected time constant of the engine alone has been computed from the dynamically determined time constants for the propeller and the engine-propeller combination by use of equation (B29) for the three altitudes for which dynamic data were available. These time constants for the engine alone are tabulated

in the following table along with the corresponding time constants as determined by the moments of inertia and the slopes of the torque-speed curves:

Pressure altitude (ft)	Corrected time constant by synthesis (sec)	Corrected time constant by moments of inertia and slopes (sec)
10,000	2.2	2.4
20,000	2.6	2.5
30,000	2.3	2.4

The time constants for the engine alone determined by synthesis of the dynamic data agree with those obtained by the steady-state slopes within the accuracy of the data.

SUMMARY OF RESULTS

The following results were obtained from an investigation of the dynamic characteristics of a turbine-propeller engine in the NACA Lewis altitude wind tunnel. For the dynamic phase of the investigation, the engine was subjected to sinusoidal fuel flows at varying frequency over a range of pressure altitudes from 10,000 to 30,000 feet.

- 1. The dynamic response of the engine-propeller combination was found to generalize for altitude over the range of frequencies investigated. The corrected time constant, which characterized the primary first-order lag of the combination had a value of approximately 1.0 second. The actual time constants were found to vary from 1.4 seconds at 10,000 feet to 3.1 seconds at 30,000 feet.
- 2. The dynamic response of the engine-propeller combination was found to be free from any gross dynamic nonlinearities for a range of fuel-flow changes corresponding to engine-speed changes representing approximately 20 percent of the engine-operating-speed range in the steady state.

- 3. The dynamic response of the combination as found by the frequency-response runs yielded time constants, that were in good agreement with those computed from the polar moments of inertia and the slopes of the steady-state-performance curves. From this agreement, the dynamic response at any altitude could be predicted from either a dynamic test or the steady-state-performance characteristics at a single altitude.
- 4. The dynamic response attributed to the propeller was found to generalize for pressure altitude and to be in good agreement with that computed from the steady-state slopes. The corrected time constant of the propeller was found to be approximately 0.36 second.
- 5. A single run yielded the dynamic response of the enginepropeller combination and of the propeller alone. From these responses, the dynamic response of the engine alone was synthesized and the corrected time constant of 2.4 seconds was found to agree with that determined from the torque-speed slopes.

Lewis Flight Propulsion Laboratory,
National Advisory Committee for Aeronautics,
Cleveland, Ohio.

APPENDIX A

SYMBOLS

The following symbols are used in this report	The	following	symbols	are	used	in	this	report
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I	polar moment of inertia (referred to engine speed), slug-ft ²
N	engine speed, rad/sec
p	differential operator, d/dt
Q	torque (referred to engine speed), lb-ft
t	time, sec
Wf	fuel flow, lb/hr
Wf,corr	corrected fuel flow, lb/hr
β	blade angle, deg
٠Δ	incremental change from steady state
δ	pressure correction factor, ambient static pressure NACA standard sea-level pressure
θ	temperature correction factor, ambient static temperature NACA standard sea-level temperature
τ	time constant, sec
ω	frequency, rad/sec
Subscripts	3 :
c	engine-propeller combination
е	engine
p	propeller
B	shaft

APPENDIX B

DERIVATION OF DYNAMIC-RESPONSE EQUATIONS

AND TIME CONSTANTS

The dynamic responses and the time constants are developed from the assumptions presented in the section "ANALYSIS". Three torques must be considered in the derivation, (1) that developed by the engine itself, (2) that of the propeller, and (3) the shaft, or measured, torque.

The engine torque is assumed to be a function of fuel flow and speed; the propeller torque is assumed to be a function of blade angle and speed.

$$Q_{\Theta} = Q_{\Theta}(W_{\Gamma}, \mathbb{N}) \tag{B1}$$

$$Q_{p} = Q_{p}(\beta, N)$$
 (B2)

More explicitly, equations (B1) and (B2) when linearized for incremental excursions about a steady-state point can be written as

$$\Delta Q_{\Theta} = \left(\frac{\partial Q_{\Theta}}{\partial M_{\mathbf{f}}}\right)_{\mathbf{N}} \Delta W_{\mathbf{f}} - \left(\frac{\partial Q_{\Theta}}{\partial N}\right)_{\mathbf{W}_{\mathbf{f}}} \Delta N \tag{B3}$$

and

$$\Delta Q_{p} = \left(\frac{\partial Q_{e}}{\partial \beta}\right)_{N} \Delta \beta + \left(\frac{\partial Q_{p}}{\partial N}\right)_{\beta} \Delta N \tag{B4}$$

where the subscripts on the partial derivatives indicate the constant . factor, and the signs are determined by the physical nature of the partial derivative.

The difference between engine torque and shaft torque accelerates the engine; therefore,

$$Q_{e} - Q_{g} = I_{e} \frac{dN}{dt}$$
 (B5)

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If only incremental changes in engine and shaft torque are considered equation (B5) may be written as

$$\Delta Q_{e} - \Delta Q_{g} = I_{e} \frac{d\Delta N}{dt}$$
 (B6)

Substitution of equation (B1) in equation (B4) yields

$$\Delta Q_{g} = \left(\frac{\partial Q_{e}}{\partial W_{f}}\right)_{N} \Delta W_{f} - \left(\frac{\partial Q_{e}}{\partial N}\right)_{W_{f}} \Delta N - I_{e} \frac{d\Delta N}{dt}$$
(B7)

At constant fuel flow, substituting p for $\frac{d}{dt}$ in equation (B7) (reference 4) and rearranging terms yield

$$\left(\frac{\nabla \delta^{\mathbf{E}}}{\nabla N}\right)^{\mathbf{A}_{\mathbf{L}}} = \frac{\left(\frac{9N}{9\delta^{\mathbf{G}}}\right)^{\mathbf{A}^{\mathbf{L}}} + \mathbf{I}^{\mathbf{G}}\mathbf{b}}{-\mathbf{I}} \tag{B8}$$

Equation (B8) represents the dynamic response of the engine alone. The time constant of the engine is given by

$$\tau_{e} = \frac{I_{e}}{\left(\frac{\partial Q_{e}}{\partial N}\right)_{W_{f}}} \tag{B9}$$

The difference between shaft torque and propeller torque accelerates the propeller. If the incremental changes in propeller and shaft torque are considered, this difference can be written as

$$\Delta Q_{g} - \Delta Q_{p} = I_{p} \frac{d\Delta N}{dt}$$
 (B10)

If equations (B4) and (B10) are combined,

$$\Delta Q_{\mathbf{g}} = \left(\frac{\partial Q_{\mathbf{p}}}{\partial \beta}\right)_{\mathbf{N}} \Delta \beta + \left(\frac{\partial Q_{\mathbf{p}}}{\partial \mathbf{N}}\right)_{\beta} \Delta \mathbf{N} + \mathbf{I}_{\mathbf{p}} \frac{\mathrm{d} \Delta \mathbf{N}}{\mathrm{d} \mathbf{t}}$$
(B11)

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which at constant blade angle can be reduced and rearranged to

$$\left(\frac{\Delta N}{\Delta Q^{B}}\right)^{B} = \frac{1}{\left(\frac{\partial Q^{D}}{\partial Q^{D}}\right)^{B} + I^{D}D}$$
(B15)

by again substituting p for d/dt. Equation (Bl2) represents the dynamic response of the propeller; the time constant of the propeller is

$$\tau_{p} = \frac{T_{p}}{\left(\frac{\partial Q_{p}}{\partial N}\right)_{\beta}} \tag{B13}$$

Equations (B7) and (B11) are expressions for shaft torque, which are equal if all torques and speeds are referred to engine speed by multiplication by the proper function of gear ratio. Therefore,

$$\left(\frac{\partial Q_{\mathbf{e}}}{\partial W_{\mathbf{f}}}\right)_{\mathbf{N}}^{\Delta W_{\mathbf{f}}} - \left(\frac{\partial Q_{\mathbf{e}}}{\partial \mathbf{N}}\right)_{\mathbf{W}_{\mathbf{f}}}^{\Delta \mathbf{N}} - \mathbf{I}_{\mathbf{e}} \frac{d\Delta \mathbf{N}}{d\mathbf{t}} = \left(\frac{\partial Q_{\mathbf{p}}}{\partial \beta}\right)_{\mathbf{N}}^{\Delta \beta} + \left(\frac{\partial Q_{\mathbf{p}}}{\partial \mathbf{N}}\right)_{\beta}^{\Delta \mathbf{N}} + \mathbf{I}_{\mathbf{p}} \frac{d\Delta \mathbf{N}}{d\mathbf{t}}$$
(B14)

At constant blade angle, by substituting p for d/dt and rearranging terms,

$$\left(\frac{\Delta N}{\Delta W_{f}}\right)_{\beta} = \frac{\left(\frac{\partial Q_{e}}{\partial W_{f}}\right)_{N}}{\left(\frac{\partial Q_{e}}{\partial W_{f}}\right)_{M} + \left(\frac{\partial Q_{p}}{\partial W_{f}}\right)_{\beta} + \left(I_{e} + I_{p}\right)p} \tag{B15}$$

Equation (Bl5) represents the dynamic response of the enginepropeller combination, with a corresponding time constant of

$$\tau_{c} = \frac{I_{\Theta} + I_{p}}{\left(\frac{\partial Q_{\Theta}}{\partial N}\right)_{W_{P}} + \left(\frac{\partial Q_{p}}{\partial N}\right)_{\beta}}$$
(B16)

If in equations (B3) to (B16) corrected values of torque (Q/ δ), speed (N/ $\overline{\theta}$), and fuel flow (W_f/ δ V $\overline{\theta}$) are used, the corrected dynamic responses and time constants will be:

For the engine alone,

$$\left(\frac{\Delta N/\sqrt{\theta}}{\Delta Q_{e}/\delta}\right)_{W_{f,corr}} = \frac{-1}{\left(\frac{\partial Q_{e}/\delta}{\partial N/\sqrt{\theta}}\right)_{W_{f,corr}} + I_{e}p}$$
(B17)

and

$$\tau_{e} \frac{\delta}{\sqrt{\theta}} = \frac{I_{e}}{\left(\frac{\partial Q_{e} / \delta}{\partial N / W_{e}}\right)_{W_{f,corr}}}$$
(B18)

For the propeller alone,

$$\left(\frac{\Delta N \sqrt{\theta}}{\Delta Q_{g} / \delta}\right)_{\beta} = \frac{1}{\left(\frac{\partial Q_{p} / \delta}{\partial N \sqrt{\theta}}\right)_{\beta} + I_{p} p}$$
(B19)

and

$$\tau_{p} \frac{\delta}{\sqrt{\theta}} = \frac{\tau_{p}}{\left(\frac{\partial Q_{p} / \delta}{\partial N / \sqrt{\theta}}\right)_{B}}$$
(B20)

and for the engine-propeller combination,

$$\left(\frac{\Delta N \sqrt{\theta}}{\Delta W_{\mathbf{f}} / \delta \sqrt{\theta}}\right)_{\beta} = \frac{\left(\frac{\partial Q_{\mathbf{g}} / \delta}{\partial W_{\mathbf{f}} / \delta \sqrt{\theta}}\right)_{N / \sqrt{\theta}}}{\left(\frac{\partial Q_{\mathbf{g}} / \delta}{\partial N / \theta}\right)_{W_{\mathbf{f}, corr}} + \left(\frac{\partial Q_{\mathbf{p}} / \delta}{\partial N / \theta}\right)_{\beta} + (I_{\mathbf{g}} + I_{\mathbf{p}})p}$$
(B21)

and

$$\tau_{c} \frac{\delta}{\sqrt{\theta}} = \frac{I_{e} + I_{p}}{\left(\frac{\partial Q_{e} / \delta}{\partial N / \sqrt{\theta}}\right)_{W_{f}, corr} + \left(\frac{\partial Q_{p} / \delta}{\partial N / \sqrt{\theta}}\right)_{\beta}}$$
(B22)

Equations (B8), (B12), (B15), (B17), (B19), and (B21) may be normalized by dividing each by the value of the response at zero frequency.

The dynamic characteristics of the engine, the propeller, or the combination can be determined from the dynamic responses of the other two. As an example of this synthesizing process, the expression for the dynamic characteristics of the engine are found from those of the propeller alone and the engine-propeller combination.

Equations (B12) and (B15) can be written as

$$\left(\frac{\nabla N}{\nabla S}\right)^{B} = \left(\frac{\partial N}{\partial S}\right)^{B} + I^{D}b = \left(\frac{\partial N}{\partial S}\right)^{B} \left(J + L^{D}b\right)$$
(BS3)

and

$$\left(\frac{\Delta W_{\underline{f}}}{\Delta N}\right)_{\beta} \left(\frac{\partial Q_{\underline{e}}}{\partial W_{\underline{f}}}\right)_{N} = \left(\frac{\partial Q_{\underline{e}}}{\partial N}\right)_{W_{\underline{f}}} + \left(\frac{\partial Q_{\underline{p}}}{\partial N}\right)_{\beta} + \left(I_{\underline{e}} + I_{\underline{p}}\right)_{\underline{p}} = \left[\left(\frac{\partial Q_{\underline{e}}}{\partial N}\right)_{W_{\underline{f}}} + \left(\frac{\partial Q_{\underline{p}}}{\partial N}\right)_{\beta}\right] \left(1 + \tau_{\underline{e}}\underline{p}\right) \tag{B24}$$

Subtracting equation (B23) from (B24)

$$\left(\frac{\Delta W_{f}}{\Delta N}\right)_{\beta} \left(\frac{\partial Q_{e}}{\partial W_{f}}\right)_{N} - \left(\frac{\Delta Q_{s}}{\Delta N}\right)_{\beta} = \left(\frac{\partial Q_{e}}{\partial N}\right)_{W_{f}} + I_{e}p$$
(B25)

or

$$\left(\frac{\Delta W_{\underline{f}}}{\Delta N}\right)_{\beta} \left(\frac{\partial Q_{\underline{e}}}{\partial W_{\underline{f}}}\right)_{N} - \left(\frac{\Delta Q_{\underline{e}}}{\Delta N}\right)_{\beta} = \left(\frac{\partial Q_{\underline{e}}}{\partial N}\right)_{W_{\underline{f}}} + \left[\left(\frac{\partial Q_{\underline{e}}}{\partial N}\right)_{W_{\underline{f}}} + \left(\frac{\partial Q_{\underline{p}}}{\partial N}\right)_{\beta}\right]_{T_{\underline{c}}} - \left(\frac{\partial Q_{\underline{p}}}{\partial N}\right)_{\beta} \uparrow_{\underline{p}}$$
(B26)

The right-hand side of equation (B25) is the negative reciprocal of the engine response (equation (B8)). Therefore, equation (B26) can be written as

$$\left(\frac{\Delta Q_{\mathbf{g}}}{\Delta N}\right)_{W_{\mathbf{f}}} = -\left(\frac{\partial Q_{\mathbf{e}}}{\partial N}\right)_{W_{\mathbf{f}}} \left(1 + \tau_{\mathbf{e}} \mathbf{p}\right)$$
(B27)

where

$$\tau_{e} = \frac{\left[\left(\frac{\partial Q_{e}}{\partial N}\right)_{W_{f}} + \left(\frac{\partial Q_{p}}{\partial N}\right)_{\beta}\right] \tau_{c} - \left(\frac{\partial Q_{p}}{\partial N}\right)_{\beta} \tau_{p}}{\left(\frac{\partial Q_{e}}{\partial N}\right)_{W_{f}}}$$
(B28)

and where T_{Θ} is the synthesized time constant of the engine. In corrected form this time constant becomes

$$\tau_{e} \frac{\delta}{\sqrt{\theta}} = \frac{\left[\left(\frac{\partial Q_{e} / \delta}{\partial N / \sqrt{\theta}} \right)_{W_{f,corr}} + \left(\frac{\partial Q_{p} / \delta}{\partial N / \sqrt{\theta}} \right)_{\beta} \right] \frac{\tau_{c} \delta}{\sqrt{\theta}} - \left(\frac{\partial Q_{p} / \delta}{\partial N / \sqrt{\theta}} \right) \frac{\tau_{p} \delta}{\sqrt{\theta}}}{\left(\frac{\partial Q_{e} / \delta}{\partial N / \sqrt{\theta}} \right)_{W_{f,corr}}}$$
(B29)

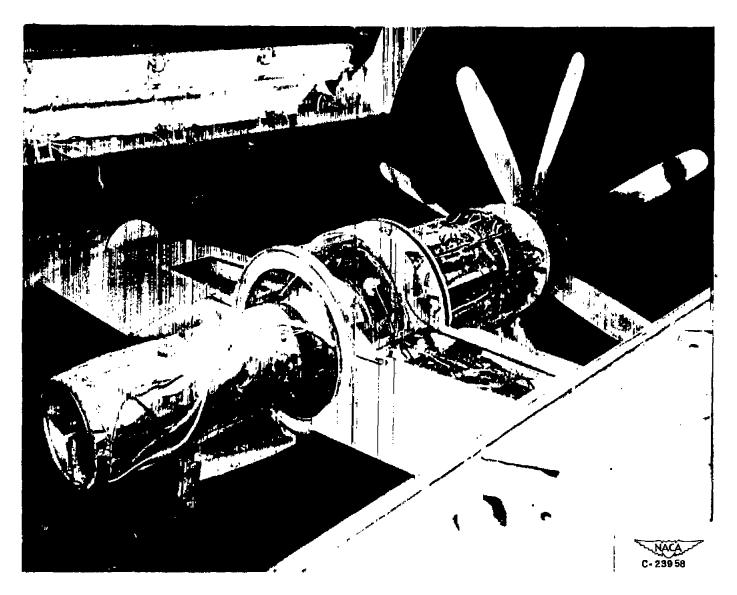
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- 4. Zemansky, Mark W.: Heat and Thermodynamics. McGraw-Hill Book Co., Inc., 2d ed., 1943, pp. 40, 72.
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. Figure 1. - Installation of turbine-propeller engine in altitude wind tunnel.

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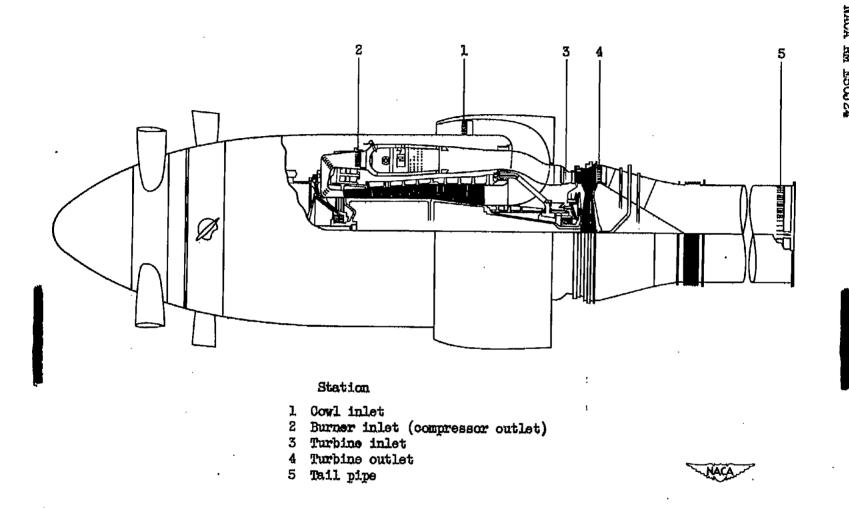


Figure 2. - Schematic diagram of turbine-propeller engine showing location of instrumentation stations.

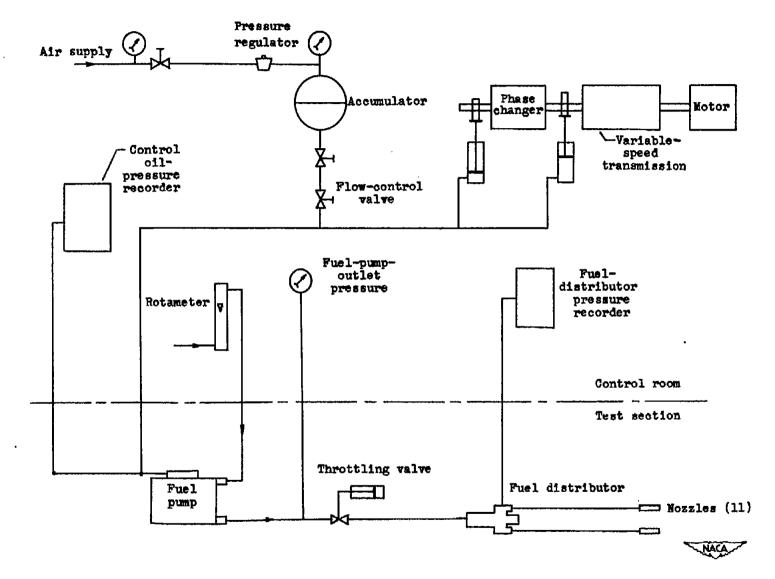
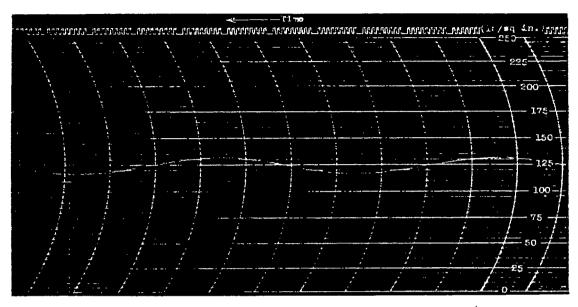


Figure 3. - Schematic diagram of fuel system used for dynamic investigation of turbine-propeller engine.



(a) Torquemeter pressure.

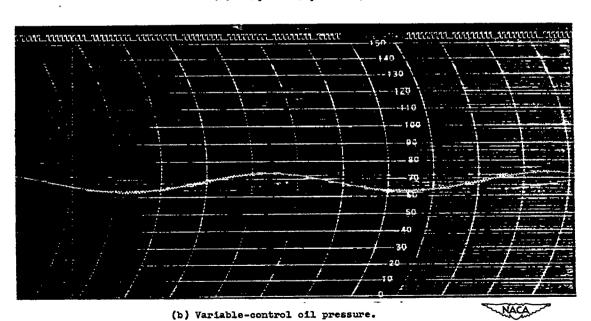
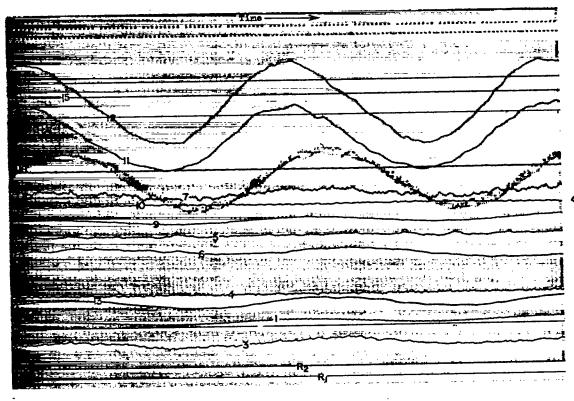


Figure 4. - Typical dynamic record traces.

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- Torque pressure
 Tail-pipe dynamic pressure
 Compressor-outlet dynamic pressure
 Compressor-inlet total pressure
 Compressor-inlet dynamic pressure
 Burner total-pressure loss
 Compressor-outlet total pressure

- Tunnel-air-stream dynamic pressure Tail-pipe temperature
- Compressor-outlet temperature 13
- Propeller pitch Engine speed 15 18
- R₁ Reference traces

(c) Oscillograph record.

Figure 4. - Concluded. Typical dynamic record traces.



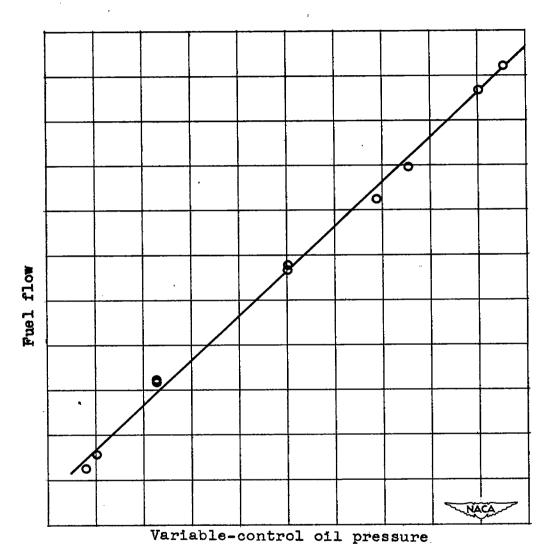


Figure 5. - Typical relation between fuel flow and variable-control oil pressure.

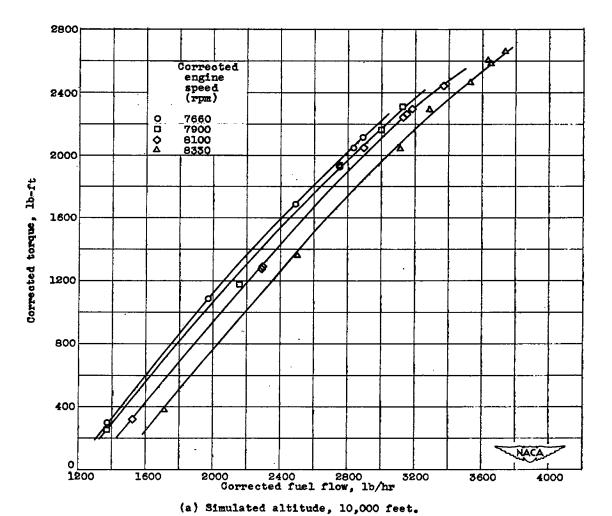
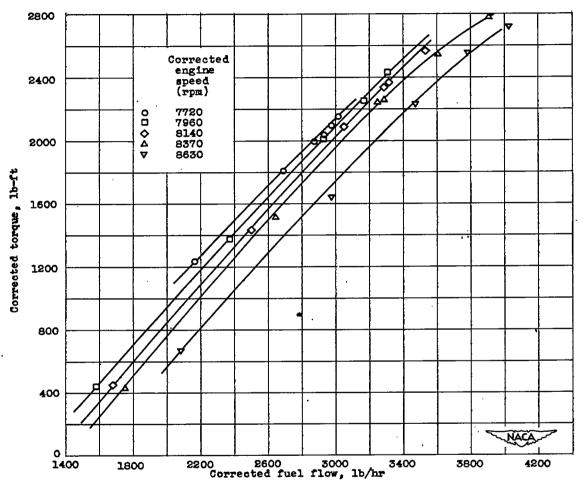


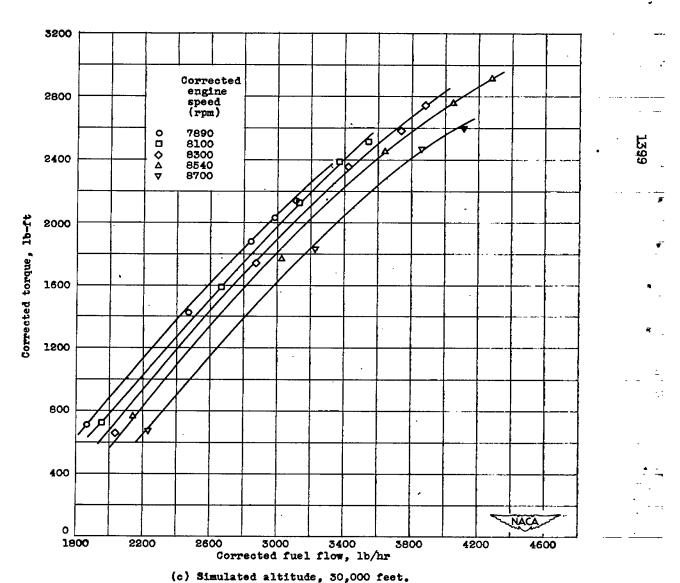
Figure 6. - Variation of corrected torque with corrected fuel flow at constant corrected engine speeds.





(b) Simulated altitude, 20,000 feet.

Figure 6. - Continued. Variation of corrected torque with corrected fuel flow at constant corrected engine speeds.



(c) Simulated altitude, 50,000 lees.

Figure 6. - Concluded. Variation of corrected torque with corrected fuel flow at constant corrected engine speeds.

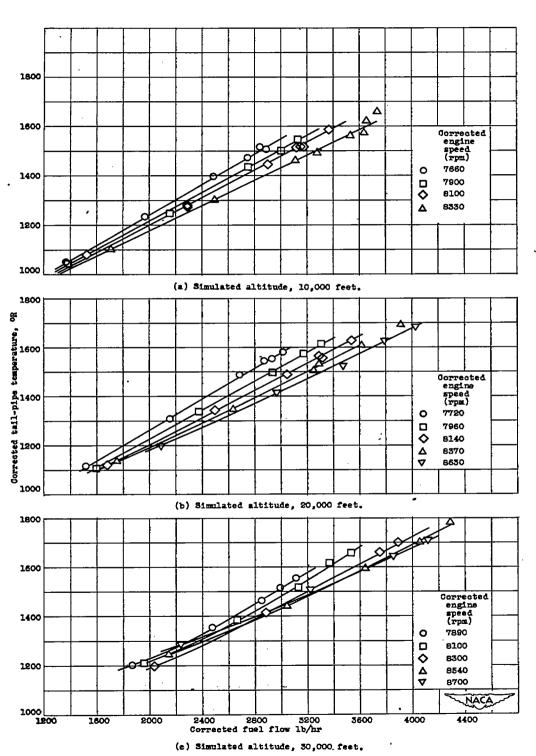


Figure 7. - Variation of corrected tail-pipe temperature with corrected fuel flow at constant corrected engine speeds.

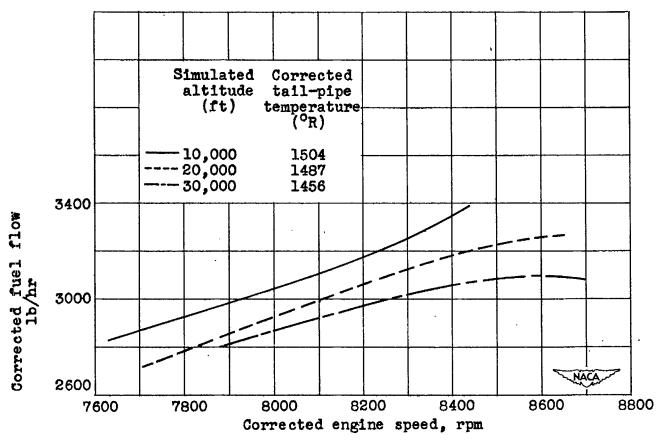


Figure 8. - Variation of corrected fuel flow with corrected engine speed at constant corrected tail-pipe temperatures.



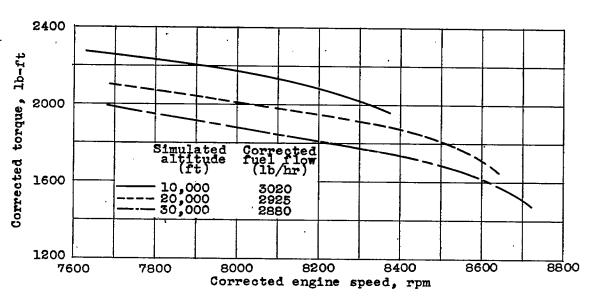


Figure 9. - Variation of corrected torque with corrected engine speed at constant corrected fuel flows.

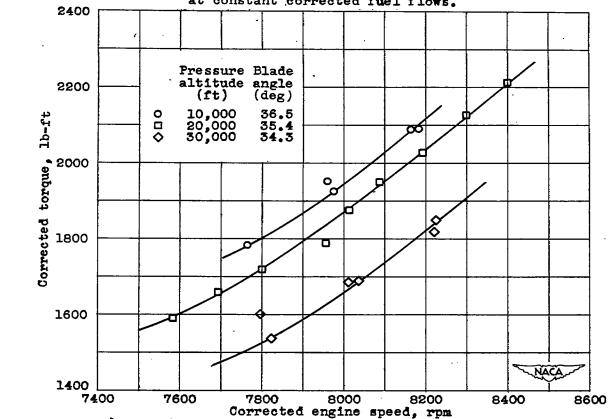


Figure 10. - Variation of corrected torque with corrected engine speed at constant blade angles.

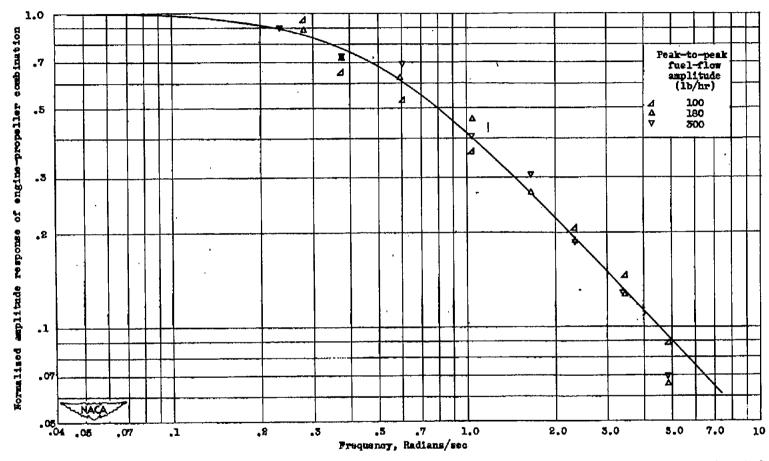


Figure 11. - Effect of imput amplitude on normalized amplitude response of engine-propeller combination, engine speed to fuel flow, at a pressure altitude of 20,000 feet.

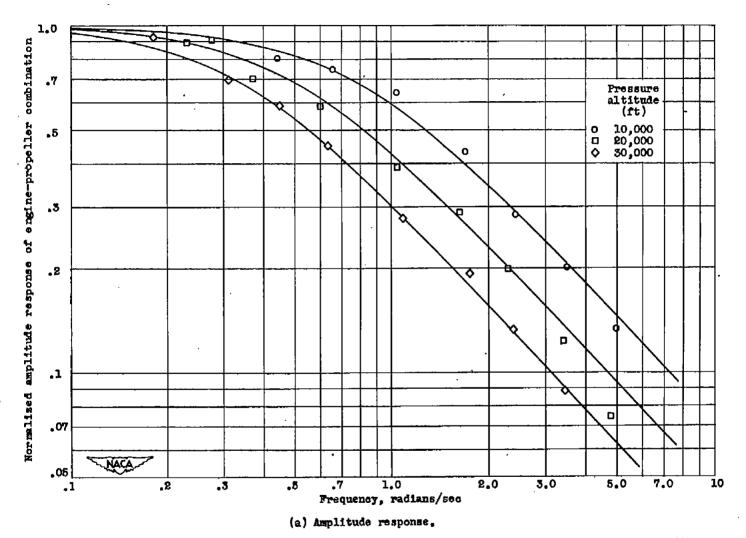


Figure 12. - Variation of dynamic response of engine-propeller combination, engine speed to fuel flow, with altitude,

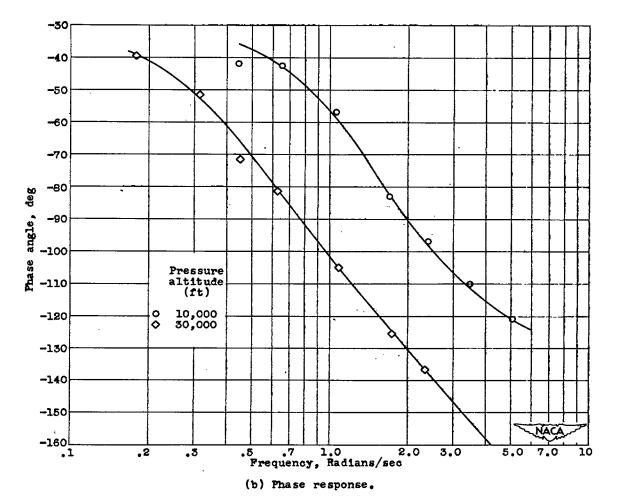
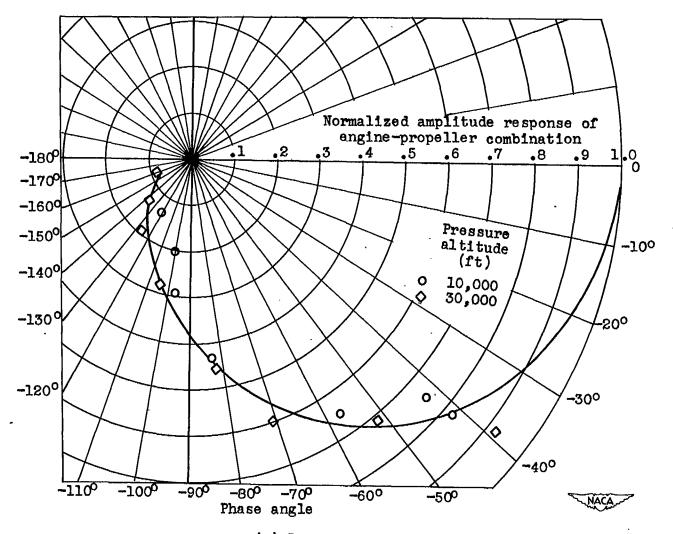


Figure 12. - Continued. Variation of dynamic response of engine-propeller combination, engine speed to fuel flow, with altitude.



(c) Locus Diagram.

Figure 12. - Concluded. Variation of dynamic response of engine-propeller combination, engine speed to fuel flow, with altitude.

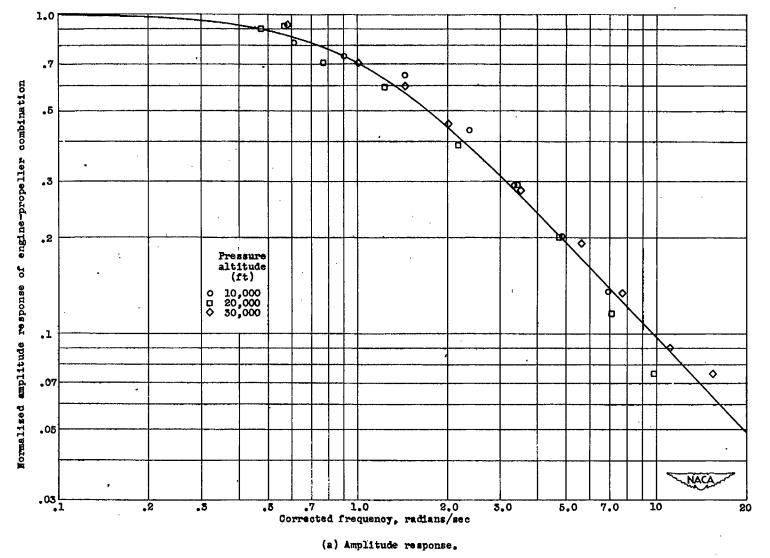
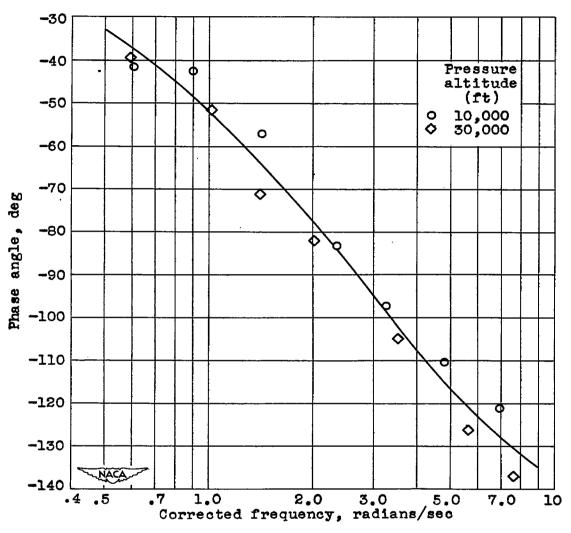


Figure 15. - Dynamic response of engine-propeller combination as function of altitude-corrected frequency.



(b) Phase response.

Figure 13. - Concluded. Dynamic response of engine-propeller combination as a function of altitude-corrected frequency.

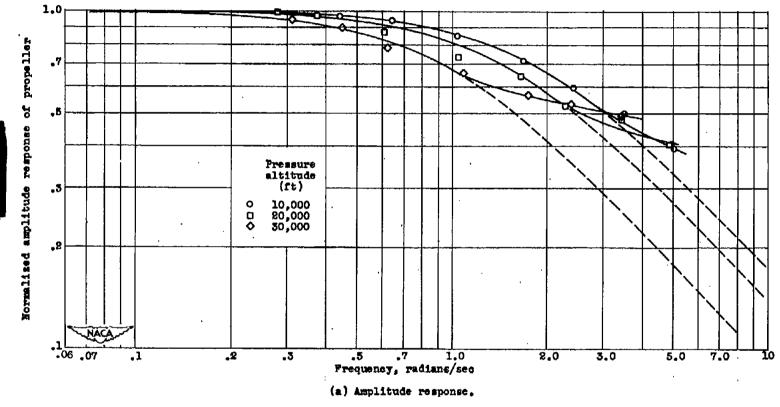


Figure 14. - Variation of dynamic response of propeller, engine speed to shaft torque, with altitude.

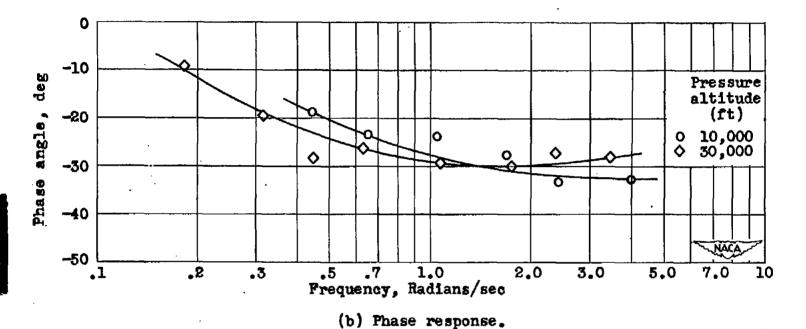


Figure 14 - Concluded. Variation of dynamic response of propeller, engine speed to shaft torque, with altitude.

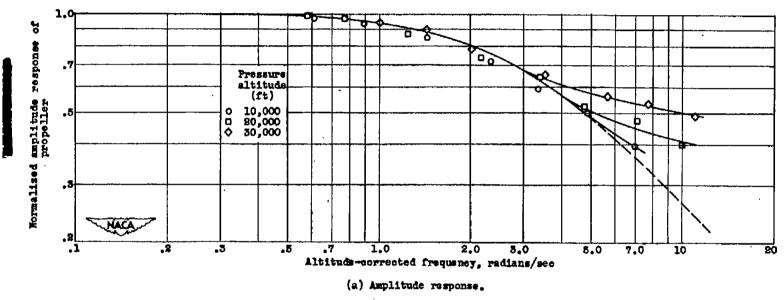


Figure 15. - Dynamic response of propeller, engine speed to shaft torque, as function of altitude-corrected frequency.

(b) Phase response.

Corrected frequency, radians/sec

Figure 15. - Concluded. Dynamic response of propeller, engine speed to shaft torque, as function of altitude-corrected frequency.



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